tance changes in excess of that predicted from the inverse-cube-root variation of the deplc-tion-layer capacitance. This is attributed to stored charge carriers under conditions of forward bias. Thus the harmonic generation efficiency of these diodes is above that predicted for cube-root capacitance variation. However, it is interesting to note that agreement between experiment and theory can be obtained for at least one published result using these diodes,[5] using Fig 4 of Hyltin and Kotzebue which is for square-root capacitance variation. The result is that of Lowell and Kiss who reported on a fifth-harmonic and eighth-harmonic generator. Using their data on the diodes used, and assuming that the effective diode capacitance at operating bias is 0.6 the zero bias values, we obtain predicted efficiencies of about 5 db for the fifth harmonic circuit and about 19 db for the eighth harmonic circuit. The values reported by Lowell and Kiss are aout 5.5 db and 19 db, respectively.

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[5] R. Lowell and M. J. Kiss, "Solid-state microwave power sources using harmonic generation," PROC. IRE, vol. 48, pp. 1334-1335; July, 1960.

Double-Layer Matching Structures*

In the millimeter wavelength region, as in other regions, the design of matching layers for dielectric surfaces is limited by the lack of suitable dielectric materials. Artificial dielectric layers, formed by periodic perturbations of the boundary surface, are not practical because of the small physical dimensions required. The following approach uses two layers of materials whose relative dielectric constants are given. The thicknesses of the layers are chosen to eliminate reflections at the desired center frequency. A broad-band match is obtained because the layers can be made less than an eighth wavelength in thickness.

In the case of normal incidence upon lossless dielectric layers, transmission line theory may be used to determine the matching conditions. Refering to Fig. 1, and assuming that the first section is matched,

$$Z_4 = Z_3 \frac{Z_2(Z_1 + jZ_2 \tan\theta_2) + jZ_3 \tan\theta_3(Z_2 + jZ_1 \tan\theta_2)}{Z_3(Z_2 + jZ_1 \tan\theta_2) + jZ_2 \tan\theta_3(Z_1 + jZ_2 \tan\theta_2)} \quad (1)$$

at the center frequency. The characteristic impedance and electrical length of the jth section are Z_j and θ_j, respectively.

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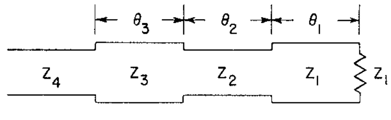

Fig. 1—Cascaded transmission line system.

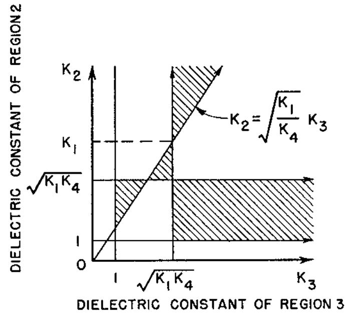

Fig. 2—Allowed values of relative dielectric constants of regions 2 and 3.

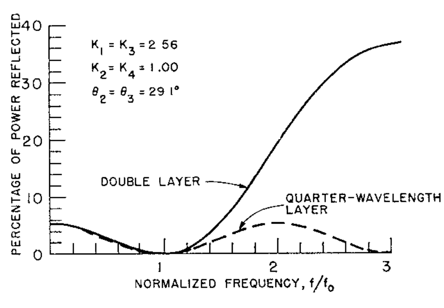

Fig. 3—Reflected power as function of normalized frequency.

Equating real and imaginary parts of (1) gives two equations which may be solved for θ_2 and θ_3

$$\theta_2 = \tan^{-1}\left[\frac{n_2^2(n_1n_4 - n_3^2)(n_4 - n_1)}{(n_2^2 - n_1n_4)(n_2^2n_4 - n_1n_3^2)}\right]^{1/2} \quad (2)$$

$$\theta_3 = \tan^{-1}\left[\frac{n_3^2(n_2^2 - n_1n_4)(n_4 - n_1)}{(n_1n_4 - n_3^2)(n_2^2n_4 - n_1n_3^2)}\right]^{1/2}. \quad (3)$$

The refractive index of the jth layer is $n_j = \sqrt{K_j}$, where K_j is the relative dielectric constant of the layer.

In a typical problem, K_1 and K_4 are specified. It may be shown that for $K_1 > K_4$, the values of K_2 and K_3 that yield real values of θ_2 and θ_3 lie in the shaded regions shown in Fig. 2. A practical design can usually be found using only available low-loss dielectric materials.

As an example, consider the problem of matching a polystyrene-air interface. Letting $K_1 = 2.56$ for polystyrene and $K_4 = 1$ for air, it is seen that one solution is

$$K_2 = 1.00$$

$$K_3 = 2.56$$

$$\theta_2 = \theta_3 = 29.1°.$$

Fig. 3 shows ratio of reflected power to incident power for this solution and for a quarter-wavelength layer designed for the same center frequency. The bandwidths of the two structures are comparable.

Since thin films of most plastics are commercially available, the double layer matching structure is feasible at millimeter wavelengths. One such structure using polystyrene film and polystyrene foam sheet on polystyrene has proved successful at 70 Gc.

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Microwave Noise-Figure Measurement for Small Noise Output*

The purpose of this communication is to propose a new method of noise-figure measurement for a microwave amplifier of small noise output. The new method is helpful when the accuracy of a conventional method is not satisfactory.

In conventional measurements, when the noise output of the microwave amplifier is too small for direct noise power measurement, an auxiliary receiver is used after the amplifier under test. The noise figure of the amplifier, F_1, is given by[1]

$$F_1 = F_{12} - \frac{F_2 - 1}{G_1}. \quad (1)$$

In this equation,

F_{12} = noise figure of over-all system,
F_2 = noise figure of the auxiliary receiver,
G_1 = gain of the amplifier under test.

It is possible to express F_1, F_2, G_1, and F_{12} in the following manner:

$$F_1 \equiv f_1 \times 10^{n_1},\ F_2 \equiv f_2 \times 10^{n_2},\ G_1 \equiv g \times 10^{m} \quad (2)$$

and

$$F_{12} \equiv f_{12} \times 10^{n_{12}}$$

where $0 < (f_1, f_2, f_{12}$ or $g) > 10$ and n_1, n_2, m and n_{12} are positive integers. Then, when $F_2 \gg 1$, (1) can be rewritten as

$$F_1 \doteq f_{12} \times 10^{n_{12}} - \frac{f_2}{g} \times 10^{n_2 - m}$$

$$= \left(f_{12} - \frac{f_2}{g} \times 10^{n_2 - m - n_{12}}\right) \times 10^{n_{12}} \quad (3)$$

* Received April 2, 1962; revised manuscript received, May 18, 1962. This is a part of research which is jointly supported by Frederick Gardner Cottell Grant and Marquette University Committee on Research Grants.

[1] H. T. Friis, "Noise figures of radio receivers," PROC. IRE, vol 32, pp. 419-422; July, 1944.

when physically realizable values are considered, it is known that

$$0 < \left(f_{12} - \frac{f_2}{g} \times 10^{n_2-m-n_{12}}\right) < 10. \quad (4)$$

Note that the accuracy of F_1 is dominated by the factor $10^{n_{12}}$ and it is difficult to obtain

$$f_{12} - \frac{f_2}{g} \times 10^{n_2-m-n_{12}}$$

accurately in conventional measuring techniques.

For example, if

$$F_1 = [5 \text{ db}] = 3.1 \times 10^0 \quad (5)$$

$$F_2 = [40 \text{ db}] = 1 \times 10^4 \quad (6)$$

and

$$G_1 = [10 \text{ db}] = 1 \times 10^1 \quad (7)$$

then

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1} = [30.013 \text{ db}] \approx [30 \text{ db}] = 1 \times 10^3. \quad (8)$$

However, when (6)–(8) are substituted back into (3)

$$F_1 \approx \left(f_{12} - \frac{f_2}{g} \times 10^{n_2-m-n_{12}}\right) \times 10^{n_{12}} = 0 \times 10^3. \quad (9)$$

Thus in practical measurement, it is very difficult to obtain $F_1 = 0.0031 \times 10^3 = 5$ db in this way.

The following new method is proposed to eliminate this kind of problem. According to the definition of noise figure[1,2]

$$F_1 = \frac{N_0}{kT_0B_1G_1} \quad (10)$$

where

N_0 = available noise output of the amplifier under test when the input is terminated by a reflectionless termination,
k = Boltzmann's constant,
T_0 = input noise temperature of the amplifier, and
B_1 = noise bandwidth of the amplifier.

If the auxiliary receiver of gain G_2, noise bandwidth B_2, and noise figure F_2 is connected after the microwave amplifier, the available noise output of the auxiliary receiver N_1 is[1]

$$\left.\begin{aligned} N_1 &= \left(N_0\frac{B_2}{B_1}\right)G_2 + (F_2 - 1)kT_0B_2G_2 \quad (\text{when } B_1 > B_2) \\ N_1 &= N_0G_2 + (F_2 - 1)kT_0B_2G_2 \quad (\text{when } B_1 < B_2). \end{aligned}\right\} \quad (11)$$

The available noise output of the auxiliary receiver alone with its input terminated by a reflectionless termination is

$$N_2 = kT_0B_2G_2F_2. \quad (12)$$

Therefore, when $B_1 > B_2$,

$$\frac{N_1}{N_2} \doteq \frac{N_0}{kT_0B_1F_2} + 1 \quad (\because F_2 \gg 1, \text{in this case})$$

or

$$N_0 = kT_0B_1F_2\left(\frac{N_1}{N_2} - 1\right). \quad (13)$$

Substituting (13) into (10) yields the result,

$$F_1 = \frac{F_2}{G_1}\cdot\left(\frac{N_1}{N_2} - 1\right). \quad (14)$$

For the case of $B_1 < B_2$,

$$\frac{N_1}{N_2} \approx \frac{N_0}{kT_0B_2F_2} + 1$$

or

$$N_0 = kT_0B_2F_2\left(\frac{N_1}{N_2} - 1\right). \quad (15)$$

Substituting (15) into (10) yields the result,

$$F_1 = \frac{F_2}{G_1}\cdot\frac{B_2}{B_1}\cdot\left(\frac{N_1}{N_2} - 1\right). \quad (16)$$

Advantages of this new method are:

1) Since this method does not include a complicated subtraction of measured values as seen in (1) but mostly multiplications and divisions, there is no problem of poor accuracy as shown in (9).
2) This method is simpler than conventional methods as seen from the comparison of (14) with (1).
3) This method does not require measurements of F_{12} which is required in the conventional method.
4) No power measurement is required. The ratio of N_1/N_2 is required which is generally easier than measuring the power. In conventional methods, if the "small signal method" was employed to measure F_{12}, the power measurement is required.

There is a limitation in this new method. This method requires measurements of B_1 and B_2 when $B_1 < B_2$. These measurements are, however, required also in the conventional method when F_{12} and F_2 are measured by the "small signal method." The conventional "noise lamp method" does not require the noise bandwidth measurement.

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[2] N. Houlding, "Noise factor," *Microwave J.*, vol. 5, pp. 74–78; January, 1962.

On Voltage and Current Continuity in the Youla-Weissfloch Circuit*

Any lossless $2n$-port can be represented as a cascade of three $2n$-ports:[1] an all-pass network, a set of n uncoupled ideal transformers, and another all-pass network. The $2n$-port may be either a physical $2n$-port [*i.e.*, a device with an even number of ports considered as a device mapping the scattering (impedance) matrix of a device connected to one half of the ports into the input scattering (impedance) matrix], or it may be a multimode transmission line. In the latter case the network may be a set of uniform lines with a discontinuity, and there may be a condition that the mode voltages or currents are continuous across the discontinuity. If a_1, a_2, b_1, and b_2 are the incident and reflected wave vectors, the voltage continuity requires $a_1+b_1=a_2+b_2$, while current continuity requires $a_1-b_1=a_2-b_2$. If the device is represented by a transfer scattering matrix,

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

the voltage continuity may be stated as:

$$[1 \;\; 1]\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [1 \;\; 1]; \quad (1a)$$

where 1 is the $n \times n$ identity matrix.

For current continuity,

$$[-1 \;\; 1]\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [1 \;\; -1]. \quad (1b)$$

Eq. (1a) requires $A+C=1$ and $B+D=1$, while (1b) would require that $C-A=1$ and $B-D=1$. The close similarity between voltage and current continuity is noted, and the detailed results that will be developed for voltage continuity will hold for current continuity with some changes of sign.

The transfer scattering matrix of the voltage continuous $2n$-port is written as:

$$T = \begin{bmatrix} A & 1-D \\ 1-A & D \end{bmatrix}. \quad (2)$$

The conditions for a lossless $2n$-port are $T^*\sigma_2T=\sigma_2$, where * is a complex conjugate transpose and

$$\sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

This leads to the following relationships:

$$A^* + A = 2, \quad (3)$$

$$A - D^* = 0, \quad (4)$$

$$D' + \bar{D} = 2. \quad (5)$$

(The primes are transposes and bars signify complex conjugates.)

* Received June 13, 1962. The work reported in this paper was performed in connection with Contracts No. AF-19(604)-7486 and AF-19(604)-4143 with the Electronic Res. Dir., Air Force Cambridge Res. Labs., Bedford, Mass.

[1] D. C. Youla, "Weissfloch equivalents for lossless $2n$-ports," IRE Trans. on Circuit Theory, vol. CT-7, pp. 193–199; September, 1960.